Math 250 – Sect.2.3: The Product and Quotient Rules, Higher Order Derivatives

I. The Product and Quotient Rules (*MEMORIZE THESE!)*

Deriving the Product Rule:

Product Rule:

Quotient Rule:

-examples- For each function, find the derivative (in simplest form)

1. $y = (3x^2 - 5)(2x + 7)$

2. $f(t) = t^2 \sin t$

3. $y = \dfrac{5x+7}{3x-4}$

4. $g(x) = \dfrac{\sin x}{\sqrt{x}}$

II. *NOW that we have the QUOTIENT RULE, we can DERIVE the differentiation formulas for the other trig functions.

-example- Find the derivative formula for $f(x) = \tan x$

-example- Derive the formula for $\dfrac{d}{dx}(\csc x)$

The other trig differentiation formulas can be derived using the same approach.

MEMORIZE:

1. $\frac{d}{dx}(\tan x) = \sec^2 x$

2. $\frac{d}{dx}(\cot x) = -\csc^2 x$

3. $\frac{d}{dx}(\sec x) = \sec x \tan x$

4. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

-examples- Find each derivative:

a. $y = 3\csc x - 5\cot x$

b. $f(w) = w^2 \sec w$

III. Higher Order Derivatives.

What do we call it when we take the DERIVATIVE of a DERIVATIVE FUNCTION?? We call it the SECOND derivative. We can take as many successive derivatives as we want!

Notation:

-example- Find the third derivative of $f(x) = 5x^4 - 3x + \cos x$

Application: In section 2.2, we examined the fact that in motion problems, the *derivative* of the position function describes the *velocity* of an object. How about the *derivative* of the *velocity* function??

POSITION:

VELOCITY:

(Speed = _____)

ACCELERATION:

-example- An automobile's velocity starting from rest is $v(t) = \sqrt{t}\cos t$, where v is measured in feet per second. Find the velocity and acceleration at t = 1, 2, 4, and 6 seconds. Discuss the SPEED of the automobile at all of those times. Is the SPEED increasing or decreasing at each of those times?